

第一章 逻辑代数基础

1. 运用基本定理证明下列等式。

$$(1) AB + \bar{A}C + \bar{B}C = AB + C$$

证明：

$$\begin{aligned} AB + \bar{A}C + \bar{B}C &= AB + (\bar{A} + \bar{B})C \\ &= AB + \overline{AB}C \\ &= AB + C \end{aligned}$$

□

$$(2) BC + D + \bar{D}(\bar{B} + \bar{C})(DA + B) = B + D$$

证明：

$$\begin{aligned} BC + D + \bar{D}(\bar{B} + \bar{C})(DA + B) &= BC + D + (\bar{B} + \bar{C})(DA + B) \\ &= BC + D + \overline{BC}(DA + B) \\ &= BC + D + DA + B \\ &= B + D \end{aligned}$$

□

$$(3) ABC + \bar{A}\bar{B}\bar{C} = \overline{\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A}}$$

证明：

$$\begin{aligned} ABC + \bar{A}\bar{B}\bar{C} &= (A + \bar{A})(A + \bar{B})(A + \bar{C})(B + \bar{A})(B + \bar{B})(B + \bar{C})(C + \bar{A})(C + \bar{B})(C + \bar{C}) \\ &= (A + \bar{B})(A + \bar{C})(B + \bar{A})(B + \bar{C})(C + \bar{A})(C + \bar{B}) \\ &= \overline{\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{A} + \bar{B}\bar{C} + \bar{C}\bar{A} + \bar{C}\bar{B}} \\ &\stackrel{\text{冗余律}}{=} \overline{\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A}} \end{aligned}$$

□

$$(4) AB + BC + CA = (A + B)(B + C)(C + A)$$

证明：

$$\begin{aligned} (A + B)(B + C)(C + A) &= ABC + ABA + ACC + ACA + BBC + BBA + BCC + BCA \\ &= ABC + AB + AC + AC + BC + BA + BC + ABC \\ &= ABC + AB + AC + BC \\ &= AB + BC + CA \end{aligned}$$

□

$$(5) \bar{A}BC + AB + A\bar{C} = BC + A\bar{C}$$

证明：

$$\begin{aligned} \bar{A}BC + AB + A\bar{C} &= B(\bar{A}C + A) + A\bar{C} \\ &= B(C + A) + A\bar{C} \\ &= BC + AB + A\bar{C} \\ &= BC + A\bar{C} \end{aligned}$$

□

$$(6) \overline{A\bar{B} + \bar{A}B} = (A + \bar{B})(\bar{A} + B)$$

证明：

$$\begin{aligned} \overline{A\bar{B} + \bar{A}B} &= \overline{A\bar{B}} \overline{\bar{A}B} \\ &= (\bar{A} + B)(A + \bar{B}) \\ &= (A + \bar{B})(\bar{A} + B) \end{aligned}$$

□

$$(7) \bar{A}\bar{B} + AB + BC = \bar{A}\bar{B} + AB + \bar{A}C$$

证明：

$$\begin{aligned} \bar{A}\bar{B} + AB + BC &= \bar{A}\bar{B} + AB + BC(A + \bar{A}) \\ &= \bar{A}\bar{B} + AB + BCA + BC\bar{A} \\ &= \bar{A}\bar{B} + AB + \bar{A}BC \\ &= \bar{A}(\bar{B} + BC) + AB \\ &= \bar{A}(\bar{B} + C) + AB \\ &= \bar{A}\bar{B} + AB + \bar{A}C \end{aligned}$$

□

2. 用逻辑代数定理化简下列逻辑函数式。

$$(1) AB + \bar{A}B\bar{C} + BC$$

$$\begin{aligned} AB + \bar{A}B\bar{C} + BC &= B(A + \bar{A}\bar{C} + C) \\ &= B(A + \bar{C} + C) \\ &= B \end{aligned}$$

$$(2) \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}BC$$

$$\begin{aligned} \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}BC &= \bar{B}(\bar{A}\bar{C} + A\bar{C} + AC) \\ &= \bar{B}(\bar{C} + AC) \\ &= \bar{B}(\bar{C} + A) \end{aligned}$$

$$(3) ab(cd + \bar{c}d)$$

$$ab(cd + \bar{c}d) = abd$$

$$(4) [x(\overline{xy})][y(\overline{xy})]$$

$$\begin{aligned} [x(\overline{xy})][y(\overline{xy})] &= xy(\overline{xy}) \overline{(xy)} \\ &= xy(\overline{xy}) \\ &= 0 \end{aligned}$$

(5) $\overline{(a+b)} \overline{(a+b)}$

$$\begin{aligned} \overline{(a+b)} \overline{(a+b)} &= \bar{a} \bar{b} \bar{a} \bar{b} \\ &= 0 \end{aligned}$$

(6) $\bar{a} \bar{b} \bar{c} + \bar{a} \bar{b} c + a \bar{b} \bar{c} + abc$

$$\begin{aligned} \bar{a} \bar{b} \bar{c} + \bar{a} \bar{b} c + a \bar{b} \bar{c} + abc &= \bar{a} \bar{b} + a \bar{b} \bar{c} + abc \\ &= \bar{b}(\bar{a} + a \bar{c}) + abc \\ &= \bar{b}(\bar{a} + \bar{c}) + abc \\ &= \bar{b} \bar{a} \bar{c} + bac \end{aligned}$$

4. 用卡诺图化简下列最小项表达式.

$$G = f(a, b, c) = \sum m(1, 3, 5, 6, 7)$$

		c	
		0	1
ab	00	0	1
	01	0	1
	11	1	1
	10	0	1

$$G = f(a, b, c) = a\bar{b} + c$$

$$H = f(w, x, y, z) = \sum m(0, 2, 8, 10)$$

		yz			
		00	01	11	10
wx	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

$$H = f(w, x, y, z) = \bar{x} \bar{z}$$

$$I = f(w, x, y, z) = \sum m(1, 3, 4, 6, 9, 12, 14, 15)$$

		yz			
		00	01	11	10
wx	00	0	1	1	0
	01	1	0	0	1
	11	1	0	1	1
	10	0	1	0	0

$$I = f(w, x, y, z) = x \oplus z \oplus (wyz)$$

$$J = f(a, b, c) = \sum m(0, 1, 2, 3, 4, 5, 7)$$

		c	
		0	1
ab	00	1	1
	01	1	1
	11	0	1
	10	1	1

$$J = f(a, b, c) = \sum M(6) = \bar{a} + b + c$$

$$K = f(a, b, c, d) = \sum m(3, 4, 5, 7, 9, 13, 14, 15)$$

		cd			
		00	01	11	10
ab	00	0	0	1	0
	01	1	1	1	0
	11	0	1	1	1
	10	0	1	0	0

$$K = f(a, b, c, d) = bd + \bar{a}b\bar{c} + \bar{a}cd + abc$$

$$L = f(a, b, c, d) = \sum m(0, 1, 2, 5, 6, 7, 8, 9, 13, 14)$$

		cd			
		00	01	11	10
ab	00	1	1	0	1
	01	0	1	1	1
	11	0	1	0	1
	10	1	1	0	0

$$L = f(a, b, c, d) = \bar{b}\bar{c} + \bar{c}d + \bar{a}bc + \bar{a}c\bar{d} + bc\bar{d}$$

5. 用卡诺图化简下列最大项表达式。

$$H = f(a, b, c, d) = \prod M(2, 3, 4, 6, 7, 10, 11, 12)$$

		cd			
		00	01	11	10
ab	00	1	1	0	0
	01	0	1	0	0
	11	0	1	1	1
	10	1	1	0	0

$$H = f(a, b, c, d) = (a + \bar{c})(\bar{b} + c + d)(\bar{a} + b + \bar{c})$$

$$F = f(u, v, w, x, y) = \prod M(0, 2, 8, 10, 16, 18, 24, 26)$$

		wx			
		00	01	11	10
uv	00	y	y	1	1
	01	y	y	1	1
	11	y	y	1	1
	10	y	y	1	1

$$F = f(u, v, w, x, y) = w + y$$

6. 化简下列带任意项的逻辑函数。

$$V = f(a, b, c, d) = \sum m(2, 3, 4, 5, 13, 15) + \sum d(8, 9, 10, 11)$$

		cd			
		00	01	11	10
ab	00	0	0	1	1
	01	1	1	0	0
	11	0	1	1	0
	10	d	d	d	d

$$V = f(a, b, c, d) = b\bar{c} + \bar{b}c$$

$$Y = f(u, v, w, x) = \sum m(1, 5, 7, 9, 13, 15) + \sum d(8, 10, 11, 14)$$

		wx			
		00	01	11	10
uv	00	0	1	0	0
	01	0	1	1	0
	11	0	1	1	d
	10	d	1	d	d

$$Y = f(u, v, w, x) = x\bar{u}\bar{v}\bar{w} = x(u + v + \bar{w})$$

$$P = f(r, s, t, u) = \sum m(0, 2, 4, 8, 10, 14) + \sum d(5, 6, 7, 12)$$

		tu			
		00	01	11	10
rs	00	1	0	0	1
	01	1	d	d	d
	11	d	0	0	1
	10	1	0	0	1

$$P = f(r, s, t, u) = \bar{u}$$

$$H = f(a, b, c, d, e) = \sum m(5, 7, 9, 12, 13, 14, 17, 19, 20, 22, 25, 27, 28, 30) + \sum d(8, 10, 24, 26)$$

		cd			
		00	01	11	10
ab	00	0	0	e	e
	01	de	d 0	\bar{e}	1
	11	de	de	\bar{e}	\bar{e}
	10	e	e	\bar{e}	\bar{e}

$$H = f(a, b, c, d, e) = a\bar{c}e + ac\bar{e} + \bar{a}\bar{b}ce + bcd\bar{e} + b\bar{c}\bar{d} + \bar{a}bc\bar{d}$$

$$I = f(d, e, f, g, h) = \prod M(5, 7, 8, 21, 23, 26, 30) \cdot \prod D(10, 14, 24, 28)$$

	fg	00	01	11	10
de	00	1	1	\bar{h}	\bar{h}
	01	h	$d1$	$d1$	1
	11	$d1$	h	h	$d1$
	10	1	1	\bar{h}	\bar{h}

$$I = f(d, e, f, g, h) = (e + \bar{f} + \bar{h})(\bar{e} + \bar{g} + h)(d + \bar{e} + f + h)$$

8. 将下列逻辑函数化简成与非形式最简式。

$$U = f(a, b, c, d) = \sum m(3, 4, 6, 11, 12, 14)$$

	cd	00	01	11	10
ab	00	0	0	1	0
	01	1	0	0	1
	11	1	0	0	1
	10	0	0	1	0

$$U = f(a, b, c, d) = b\bar{d} + \bar{b}cd = \overline{\overline{b\bar{d}} \overline{\bar{b}cd}}$$

$$V = f(a, b, c, d) = \sum m(0, 1, 2, 5, 8, 10, 13)$$

		cd			
		00	01	11	10
ab	00	1	1	0	1
	01	0	1	0	0
	11	1	0	0	1
	10	0	1	0	0

$$\begin{aligned} V = f(a, b, c, d) &= (\overline{a \oplus b \oplus d} + \overline{a \bar{b}}) \overline{cd} = \overline{a \oplus b \oplus d \bar{a} \bar{b} c d} = \overline{(\bar{a}b + a\bar{b}) \oplus d \bar{c} d} \\ &= \overline{(\bar{a}b + a\bar{b})d} \overline{(\bar{a}b + a\bar{b})\bar{d} \bar{c} d} = \overline{\bar{a}b a \bar{b} d} \overline{\bar{a}b a \bar{b} \bar{d} \bar{c} d} \end{aligned}$$

$$W = f(a, b, c, d) = \sum m(3, 5, 7, 10, 11)$$

		cd			
		00	01	11	10
ab	00	0	0	1	0
	01	0	1	1	0
	11	0	0	0	0
	10	0	1	0	1

$$W = f(a, b, c, d) = \bar{a}cd + \bar{a}bd + a\bar{b}\bar{c}d + a\bar{b}c\bar{d} = \overline{\bar{a}c\bar{d}} \overline{\bar{a}b\bar{d}} \overline{a\bar{b}\bar{c}d} \overline{a\bar{b}c\bar{d}}$$

9. 将下列逻辑函数化简成或非形式最简式。

$$G = f(a, b, c, d) = \prod M(0, 1, 2, 5, 8, 10, 13)$$

ab \ cd	00	01	11	10
00	0	0	1	0
01	1	0	1	1
11	1	0	1	1
10	0	1	1	0

$$G = f(a, b, c, d) = (b + d)(c + \bar{d} + a\bar{b}\bar{c}) = \overline{\overline{b + d} \overline{c + \bar{d} + a\bar{b}\bar{c}}} = \overline{\overline{b + d} \overline{c + \bar{d} + \bar{a} + b + c}}$$

$$H = f(a, b, c, d) = \prod M(3, 5, 7, 9, 11)$$

ab \ cd	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	1	1	1	1
10	1	0	0	1

$$H = f(a, b, c, d) = \bar{d} + \bar{a}\bar{b}\bar{c} + ab = \bar{d} + \overline{a + b + c} + \bar{a} + \bar{b}$$