# 第一章　逻辑代数基础

1. 运用基本定理证明下列等式。

(1) $AB + \overline{A}C + \overline{B}C = AB + C$

证明：

$$\begin{aligned} AB + \overline{A}C + \overline{B}C &= AB + (\overline{A} + \overline{B})C \\ &= AB + \overline{A\overline{B}}C \\ &= AB + C \end{aligned}$$

□

(2) $BC + D + \overline{D}(\overline{B} + \overline{C})(DA + B) = B + D$

证明：

$$\begin{aligned} BC + D + \overline{D}(\overline{B} + \overline{C})(DA + B) &= BC + D + (\overline{B} + \overline{C})(DA + B) \\ &= BC + D + \overline{BC}(DA + B) \\ &= BC + D + DA + B \\ &= B + D \end{aligned}$$

□

(3) $ABC + \overline{A}\,\overline{B}\,\overline{C} = \overline{A\overline{B} + B\overline{C} + C\overline{A}}$

证明：

$$\begin{aligned} ABC + \overline{A}\,\overline{B}\,\overline{C} &= (A + \overline{A})(A + \overline{B})(A + \overline{C})(B + \overline{A})(B + \overline{B})(B + \overline{C})(C + \overline{A})(C + \overline{B})(C + \overline{C}) \\ &= (A + \overline{B})(A + \overline{C})(B + \overline{A})(B + \overline{C})(C + \overline{A})(C + \overline{B}) \\ &= \overline{A\overline{B} + A\overline{C} + B\overline{A} + B\overline{C} + C\overline{A} + C\overline{B}} \\ &\xlongequal{\text{冗余律}} \overline{A\overline{B} + B\overline{C} + C\overline{A}} \end{aligned}$$

□

(4) $AB + BC + CA = (A + B)(B + C)(C + A)$

证明：

$$\begin{aligned} (A + B)(B + C)(C + A) &= ABC + ABA + ACC + ACA + BBC + BBA + BCC + BCA \\ &= ABC + AB + AC + AC + BC + BA + BC + ABC \\ &= ABC + AB + AC + BC \\ &= AB + BC + CA \end{aligned}$$

□

(5) $\overline{A}BC + AB + A\overline{C} = BC + A\overline{C}$

证明：

$$\begin{aligned}\overline{A}BC + AB + A\overline{C} &= B(\overline{A}C + A) + A\overline{C} \\ &= B(C + A) + A\overline{C} \\ &= BC + AB + A\overline{C} \\ &= BC + A\overline{C}\end{aligned}$$

□

(6) $\overline{A\overline{B} + \overline{A}B} = (A + \overline{B})(\overline{A} + B)$

证明：

$$\begin{aligned}\overline{A\overline{B} + \overline{A}B} &= \overline{A\overline{B}}\ \overline{\overline{A}B} \\ &= (\overline{A} + B)(A + \overline{B}) \\ &= (A + \overline{B})(\overline{A} + B)\end{aligned}$$

□

(7) $\overline{A}\,\overline{B} + AB + BC = \overline{A}\,\overline{B} + AB + \overline{A}C$

证明：

$$\begin{aligned}\overline{A}\,\overline{B} + AB + BC &= \overline{A}\,\overline{B} + AB + BC(A + \overline{A}) \\ &= \overline{A}\,\overline{B} + AB + BCA + BC\overline{A} \\ &= \overline{A}\,\overline{B} + AB + \overline{A}BC \\ &= \overline{A}(\overline{B} + BC) + AB \\ &= \overline{A}(\overline{B} + C) + AB \\ &= \overline{A}\,\overline{B} + AB + \overline{A}C\end{aligned}$$

□

2. 用逻辑代数定理化简下列逻辑函数式。

(1) $AB + \overline{A}B\overline{C} + BC$

$$\begin{aligned}AB + \overline{A}B\overline{C} + BC &= B(A + \overline{A}\,\overline{C} + C) \\ &= B(A + \overline{C} + C) \\ &= B\end{aligned}$$

(2) $\overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$

$$\begin{aligned}\overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C &= \overline{B}(\overline{A}\,\overline{C} + A\overline{C} + AC) \\ &= \overline{B}(\overline{C} + AC) \\ &= \overline{B}(\overline{C} + A)\end{aligned}$$

(3) $ab(cd + \bar{c}\,d)$

$$ab(cd + \bar{c}\,d) = abd$$

(4) $[x\overline{(xy)}][y\overline{(xy)}]$

$$\begin{aligned}[x\overline{(xy)}][y\overline{(xy)}] &= xy\overline{(xy)}\ \overline{(xy)} \\ &= xy\overline{(xy)} \\ &= 0\end{aligned}$$

(5) $\overline{(a+b)}\ \overline{(\bar{a}+\bar{b})}$

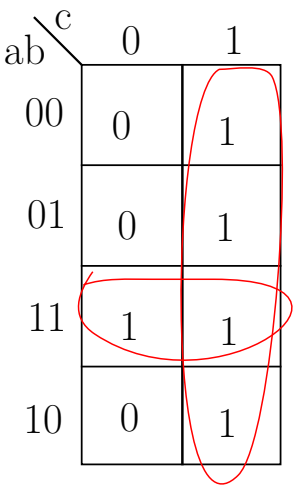
$$\overline{(a+b)}\ \overline{(\bar{a}+\bar{b})} = \bar{a}\,\bar{b}ab$$
$$= 0$$

(6) $\bar{a}\,\bar{b}\,\bar{c} + \bar{a}\,\bar{b}c + a\bar{b}\,\bar{c} + abc$

$$\bar{a}\,\bar{b}\,\bar{c} + \bar{a}\,\bar{b}c + a\bar{b}\,\bar{c} + abc = \bar{a}\,\bar{b} + a\bar{b}\,\bar{c} + abc$$
$$= \bar{b}(\bar{a} + a\bar{c}) + abc$$
$$= \bar{b}(\bar{a} + \bar{c}) + abc$$
$$= \bar{b}\overline{ac} + bac$$

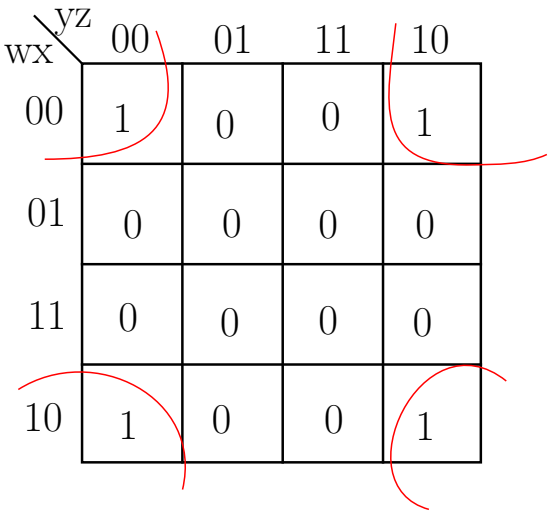
4. 用卡诺图化简下列最小项表达式.

$G = f(a,b,c) = \sum m(1,3,5,6,7)$



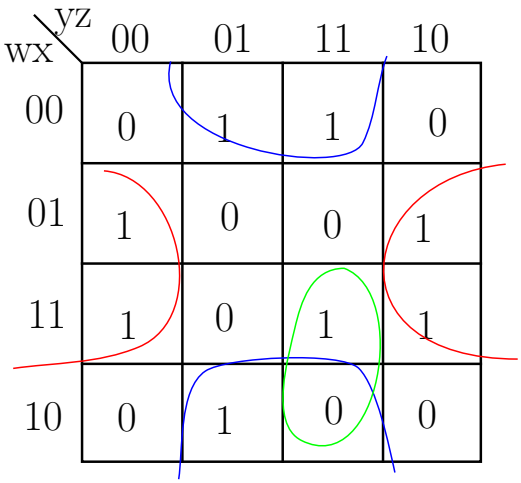

$$G = f(a,b,c) = a\bar{b} + c$$

$H = f(w,x,y,z) = \sum m(0,2,8,10)$



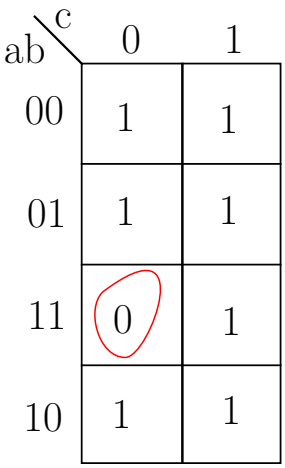

$$H = f(w,x,y,z) = \bar{x}\,\bar{z}$$

$I = f(w,x,y,z) = \sum m(1,3,4,6,9,12,14,15)$




$I = f(w,x,y,z) = x \oplus z \oplus (wyz)$

$J = f(a,b,c) = \sum m(0,1,2,3,4,5,7)$




$J = f(a,b,c) = \sum M(6) = \bar{a} + b + c$

$K = f(a,b,c,d) = \sum m(3,4,5,7,9,13,14,15)$

| ab \ cd | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 0 | 1 | 1 | 1 |
| 10 | 0 | 1 | 0 | 0 |

$$K = f(a,b,c,d) = bd + \bar{a}b\bar{c} + \bar{a}cd + abc$$

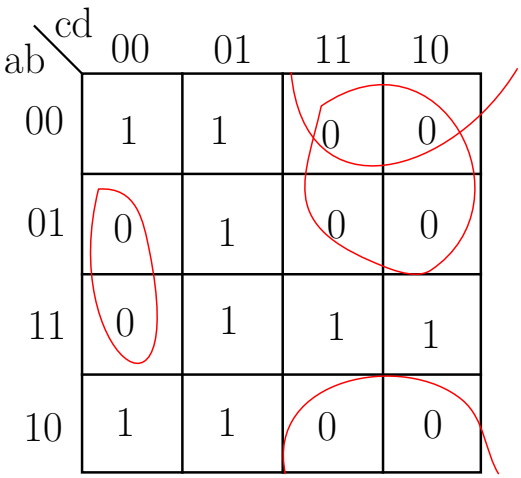
$L = f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,13,14)$

| ab \ cd | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 0 | 1 |
| 01 | 0 | 1 | 1 | 1 |
| 11 | 0 | 1 | 0 | 1 |
| 10 | 1 | 1 | 0 | 0 |

$$L = f(a,b,c,d) = \bar{b}\,\bar{c} + \bar{c}d + \bar{a}bc + \bar{a}c\bar{d} + bc\bar{d}$$

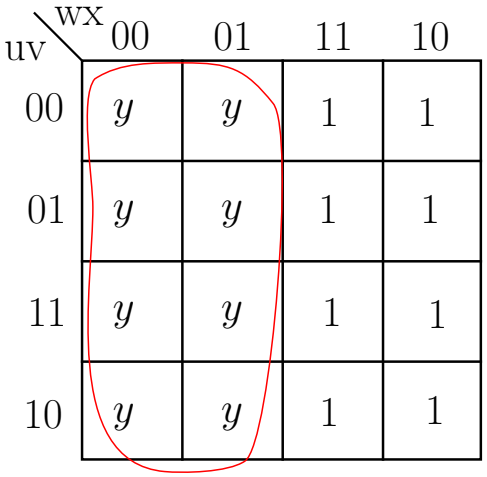
5. 用卡诺图化简下列最大项表达式。

$H = f(a,b,c,d) = \prod M(2,3,4,6,7,10,11,12)$




$$H = f(a,b,c,d) = (a+\bar{c})(\bar{b}+c+d)(\bar{a}+b+\bar{c})$$

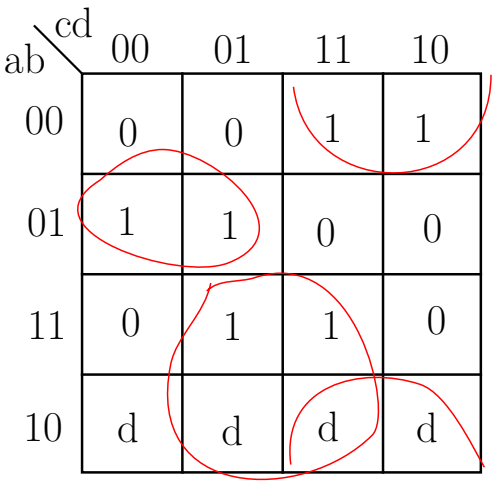
$F = f(u,v,w,x,y) = \prod M(0,2,8,10,16,18,24,26)$




$$F = f(u,v,w,x,y) = w + y$$

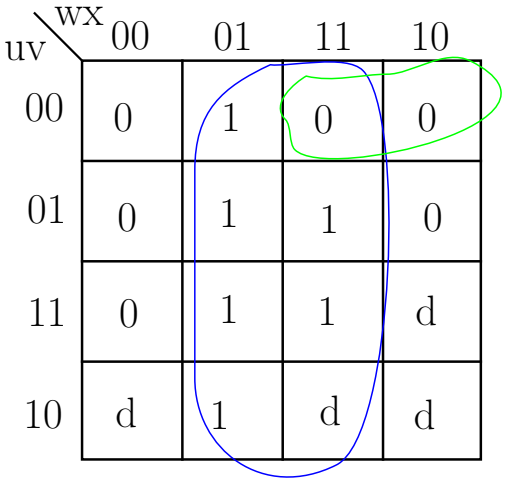
6. 化简下列带任意项的逻辑函数。

$V = f(a,b,c,d) = \sum m(2,3,4,5,13,15) + \sum d(8,9,10,11)$




$$V = f(a,b,c,d) = b\bar{c} + \bar{b}c$$

$Y = f(u,v,w,x) = \sum m(1,5,7,9,13,15) + \sum d(8,10,11,14)$




$$Y = f(u,v,w,x) = x\overline{\bar{u}\,\bar{v}\,w} = x(u + v + \bar{w})$$

$P = f(r,s,t,u) = \sum m(0,2,4,8,10,14) + \sum d(5,6,7,12)$

| rs \ tu | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | 1 | d | d | d |
| 11 | d | 0 | 0 | 1 |
| 10 | 1 | 0 | 0 | 1 |

$P = f(r,s,t,u) = \bar{u}$

$H = f(a,b,c,d,e) = \sum m(5,7,9,12,13,14,17,19,20,22,25,27,28,30) + \sum d(8,10,24,26)$

| ab \ cd | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | $e$ | $e$ |
| 01 | d$e$ | d 0 | $\bar{e}$ | 1 |
| 11 | d$e$ | d$e$ | $\bar{e}$ | $\bar{e}$ |
| 10 | $e$ | $e$ | $\bar{e}$ | $\bar{e}$ |

$H = f(a,b,c,d,e) = a\bar{c}e + ac\bar{e} + \bar{a}\,\bar{b}ce + bcd\bar{e} + b\bar{c}\,\bar{d} + \bar{a}bc\bar{d}$

$I = f(d,e,f,g,h) = \prod M(5,7,8,21,23,26,30) \cdot \prod D(10,14,24,28)$

| de \ fg | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | $\bar{h}$ | $\bar{h}$ |
| 01 | $h$ | d 1 | d 1 | 1 |
| 11 | d 1 | $h$ | $h$ | d 1 |
| 10 | 1 | 1 | $\bar{h}$ | $\bar{h}$ |

$I = f(d,e,f,g,h) = (e + \bar{f} + \bar{h})(\bar{e} + \bar{g} + h)(d + \bar{e} + f + h)$

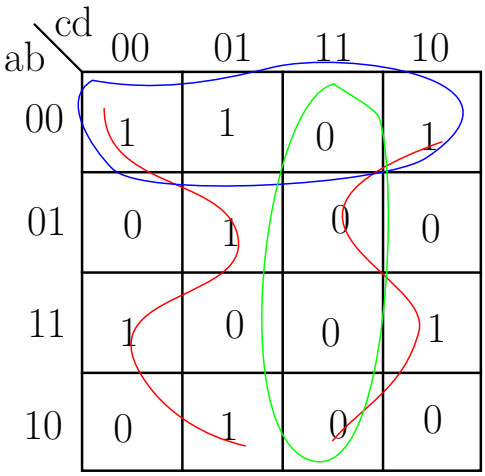
8. 将下列逻辑函数化简成与非形式最简式。

$U = f(a,b,c,d) = \sum m(3,4,6,11,12,14)$

| ab \ cd | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | 0 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 1 | 0 | 0 | 1 |
| 10 | 0 | 0 | 1 | 0 |

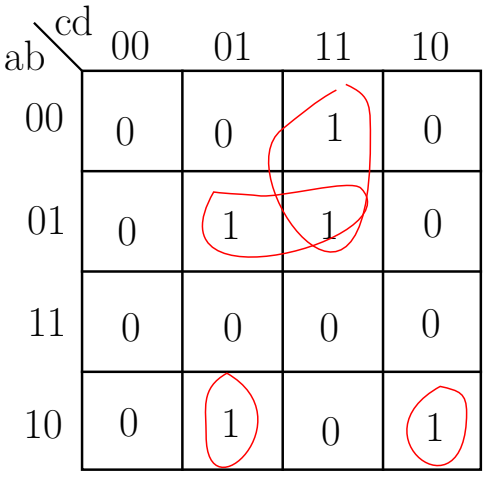
$U = f(a,b,c,d) = b\bar{d} + \bar{b}cd = \overline{\overline{b\bar{d}}\ \overline{\bar{b}cd}}$

$V = f(a,b,c,d) = \sum m(0,1,2,5,8,10,13)$




$$V = f(a,b,c,d) = (\overline{a \oplus b \oplus d} + \bar{a}\,\bar{b})\overline{cd} = \overline{a \oplus b \oplus d}\,\overline{\bar{a}\,\bar{b}}\,\overline{cd} = \overline{(\bar{a}\,b + a\,\bar{b}) \oplus d}\ \overline{cd}$$
$$= \overline{\overline{(\bar{a}\,b + a\,\bar{b})}\,d}\ \overline{(\bar{a}\,b + a\,\bar{b})\,\bar{d}}\ \overline{cd} = \overline{\bar{a}\,\bar{b}\,a\,\bar{b}\,d}\ \overline{\overline{\bar{a}\,\bar{b}\,a\,\bar{b}}\,\bar{d}}\ \overline{cd}$$

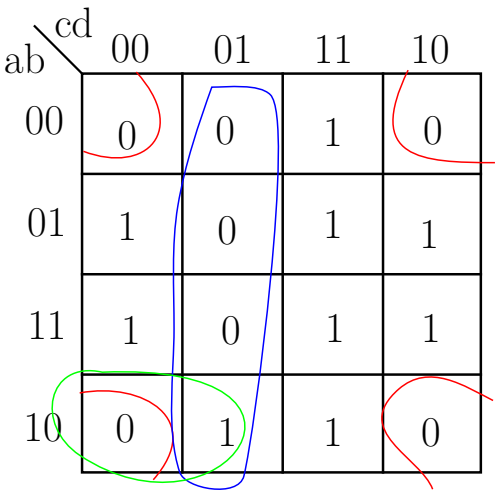
$W = f(a,b,c,d) = \sum m(3,5,7,10,11)$




$$W = f(a,b,c,d) = \bar{a}\,cd + \bar{a}\,bd + a\,\bar{b}\,\bar{c}\,d + a\,\bar{b}\,c\,\bar{d} = \overline{\bar{a}\,cd}\ \overline{\bar{a}\,bd}\ \overline{a\,\bar{b}\,\bar{c}\,d}\ \overline{a\,\bar{b}\,c\,\bar{d}}$$

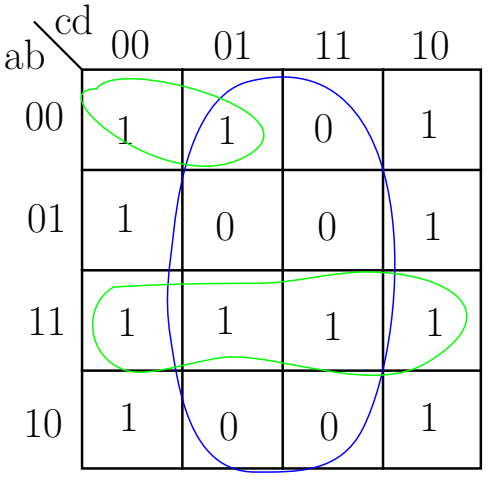
9. 将下列逻辑函数化简成或非形式最简式。

$G = f(a,b,c,d) = \prod M(0,1,2,5,8,10,13)$




$$G = f(a,b,c,d) = (b+d)(c+\bar{d}+a\bar{b}\,\bar{c}) = \overline{\overline{b+d}\;\overline{c+\bar{d}+\overline{\bar{a}+b+c}}}$$

$H = f(a,b,c,d) = \prod M(3,5,7,9,11)$




$$H = f(a,b,c,d) = \bar{d} + \bar{a}\,\bar{b}\,\bar{c} + ab = \bar{d} + \overline{a+b+c} + \overline{\bar{a}+\bar{b}}$$